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DETERMINATION OF THE EFFECT OF THE WALLS OF A WIND-TUNNEL FROM THE PARAMETERS OF FLOW NEAR THEM

[OPREDELENIE VLIYANIYA STENOK AERODINAMICHESKOI TRUBY PO PARAMETRAM POTOKA VBLIZI NIKH]

by

S. A. Glaskov

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EDITOR'S SUMMARY

This paper describes a method based on the use of two flow variables measured at a boundary close to the tunnel wall to calculate wall interference. It is similar to earlier work in the West although the author was probably unaware of this work.

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The methods for estimating interference and corrections due to the walls of a tunnel have received a new stimulus for development in the last decade[1,2]. The classical theories of wall interference used linear homogeneous boundary conditions to describe the behaviour of the characteristics of perforated walls. The disadvantage of this approximation is that the magnitude of the porosity of the walls, which is necessary for the formulation of the boundary conditions, is not known *a priori* and usually varies with variation of the configuration of the wall, the M number, the Re number, the geometry of the model and the load of the tunnel[3].

A method for estimating wall interference from the measured parameters of the flow on the control surface near the boundaries was first proposed in paper (4) for a wing of infinite span without lift in a two-dimensional flow. In paper (5) the method was extended to axisymmetric bodies in a tunnel having porous walls. Satisfactory results in estimating wall corrections to the pressure were obtained up to M = 0.8. In paper (6) the problem was solved for two-dimensional flow about a wing of infinite span at an angle of attack in an analogous formulation.

In the classical formulation with linear boundary conditions, expressions have been obtained by the method of images which determine the downwash of the flow (in the region of the location of the model), caused by the walls of the tunnel for a wing with lift, papers (7,9). The simplicity and convenience of this method are obvious. But it is doubtful whether the natural difficulties which arise when solving the problem of determining wall interference with a varying parameter of porosity on the surface can be overcome in a three-dimensional flow using the method of images.

In papers (8,9) the flow characteristics of perforated panels have been investigated, and a fairly simple method has been developed for determining the parameters of porosity of the walls. Static pressure orifices over the whole surface of the working section of the tunnel make it possible to measure the distribution of the static pressure, and hence the axial perturbed component of velocity. From the well-known parameter for porosity (linking the pressure on the wall from the side of the flow and the downwash angle) it is possible to determine the component of velocity which is perpendicular to the wall.

If the procedure for measuring the two components on the control surface near the walls is followed, there is a real possibility of estimating wall-induced velocities by the proposed method.

1 Let us consider that the model of the wing located in the tunnel at a small angle of attack in a subsonic steady non-viscous gas flow. Let us examine the problem to the order of approximation of linear theory (see Fig 1).

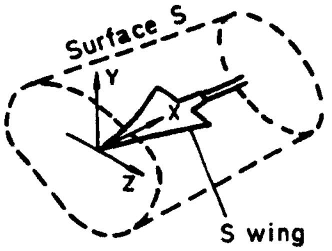

Fig 1

In the transformed coordinates (Fig 1)

$$x = \frac{X}{c}, \quad y = \frac{Y}{c}, \quad z = \frac{r}{c},$$

where c is the maximum chord of the wing, $\beta = \sqrt{1 - M_\infty^2}$ is the Prandtl-Glauert factor, and, the flow is described by the Laplace equation for the potential of perturbed velocity $\Phi(x,y,z)$

$$\nabla^2\Phi(x,y,z) = 0 .$$

The condition of no-flow across the wing surface, referred to the plane (Y = 0), assumes the form

$$\left.\frac{\partial\Phi(x,\pm 0,z)}{\partial y}\right|_{(x,z)(\in S_{wing})} = \frac{1}{\beta}(f'_x(x,2) \pm h'_x(x,z)) ,$$

where $f(x,y)$ is the equation of the shape of the mean line of the wing, $h(x,y)$ is the thickness of the wing relative to the mean line, S_{wing} is the surface of the wing, $|h,f| \ll 1$.

Let us define the required potential of the perturbed flow Φ as the sum

$$\Phi(x,y,z) = \Phi_S(x,y,z) + \Phi_A(x,y,z) ,$$

where $\Phi_S(x,y,z)$ and $\Phi_A(x,y,z)$ are the symmetrical and anti-symmetrical functions relative to the plane xz , y = 0 respectively. From the definition

$$U = \frac{\partial\Phi}{\partial x}, \quad V = \frac{\partial\Phi}{\partial y}, \quad W = \frac{\partial\Phi}{\partial z} .$$

2 The symmetrical potential $\Phi_S(x,y,z)$ satisfies the Laplace equation

$$\nabla^2\Phi_S(x,y,z) = 0 ,$$

and the boundary condition on the body

$$\left.\frac{\partial\Phi(x,\pm 0z)}{\partial y}\right|_{(x,z)(\in S_{wing})} = \pm\frac{h'_x(x,z)}{\beta} .$$

On the surface S (Fig 1) $\frac{\partial\Phi_S}{\partial x}$ and $\frac{\partial\Phi_S}{\partial n}$ are known.

From Green's theorem the following is correct for the potential Φ_S

$$\Phi_S^T(x,y,z) = \frac{1}{4\pi}\iint\limits_{S_{wing}} \left\{ \Phi_s(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_s(\xi,\eta,\zeta)}{\partial n}\frac{1}{r}\right\} d\xi\, d\zeta +$$

$$+\frac{1}{4\pi}\iint\limits_{S} \left\{ \Phi_s(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_s(\xi,\eta,\zeta)}{\partial n}\cdot\frac{1}{r}\right\} dS , \qquad (1)$$

$r = [(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{1/2}$; the suffix T indicates that the flow is bounded by the surface, S . (See Fig 2.)

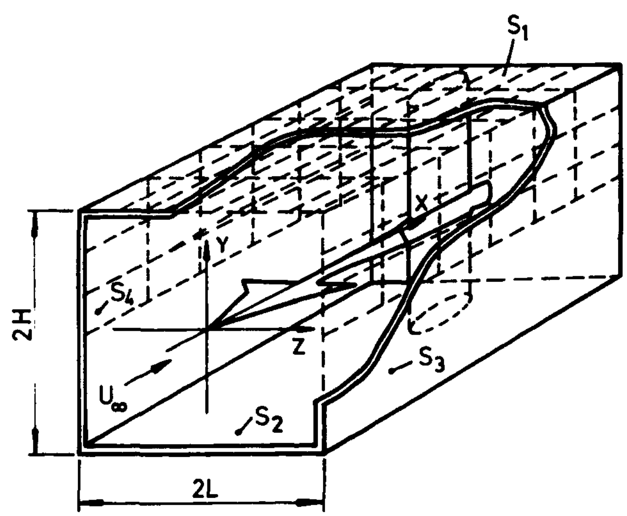

Fig 2

o

The integral for the surface S_{wing} is, in accordance with symmetry (and by removal of the boundary condition to the plane), transformed to the form:

$$\frac{1}{4\pi}\iint\limits_{S_{wing}} \left\{ \Phi_s(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\cdot\frac{1}{r}\right) - \frac{\partial\Phi(\xi,\eta,\zeta)}{\partial n}\frac{1}{r} \right\} d\xi d\zeta = \frac{1}{2\pi}\iint\limits_{S_{wing}} \frac{\partial\Phi(\xi,\eta,\zeta)}{\partial\eta}\frac{d\xi d\zeta}{r}. \quad (2)$$

The integrals for the surface S vanish in the case of unbounded flow. Thus the potential of such a flow

$$\Phi_S^F(x,y,z) = \frac{1}{2\pi}\iint\limits_{S_{wing}} \frac{\partial\Phi(\xi,0,\zeta)}{\partial\zeta}\frac{d\xi d\zeta}{r}. \quad (3)$$

Taking (1) to (3) into account, we can write the expression for the wall-induced potential

$$\Phi_s^i(x,y,z) = \Phi_S^T(x,y,z) - \Phi_S^F(x,y,z) =$$
$$= \frac{1}{4\pi}\iint\limits_{S} \left\{ \Phi_S^T(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_S^T(\xi,\eta,\zeta)}{\partial n}\frac{1}{r} \right\} dS .$$

In a wind-tunnel having a rectangular working section (Fig 2) we obtain for the induced component of velocity along the axis X in the plane of location of the model (taking symmetry into account)

$$U_s^i(x,0,z) = \frac{\partial\Phi i(x,0,z)}{\partial x} =$$
$$= \frac{1}{2\pi}\iint\limits_{S_1} \left\{ \frac{H \cdot U_s(\xi,H,\zeta) - V_s(\xi,H,\zeta)(x-\xi)}{[(x-\xi)^2 + H^2 + (z-\zeta)^2]^{3/2}} \right\} d\xi d\zeta +$$
$$+ \frac{1}{2\pi}\iint\limits_{\frac{1}{2}S_3} \left\{ \frac{-U_s^T(\xi,\eta,L)(z-L) - W_s^T(\xi,\eta,L)(x-\xi)}{[(x-\xi)^2 + \eta^2 + (z-L)^2]^{3/2}} \right\} d\xi d\eta +$$
$$+ \frac{1}{2\pi}\iint\limits_{\frac{1}{2}S_4} \left\{ \frac{U_s^T(\xi,\eta,L)(z+L) - W_s^T(\xi,\eta,L)(x-\xi)}{[(x-\xi)^2 + \eta^2 + (z+L)^2]^{3/2}} \right\} d\xi d\eta , \quad (4)$$

where S_1 is the surface of the upper wall, $\frac{1}{2}S_3$ is the upper half of the right-hand wall, and $\frac{1}{2}S_4$ is the upper half of the left-hand wall.

The symmetrical components of velocity on the surface of the tunnel are determined as follows:

$$U_S^T(\xi,\eta,\zeta) = \tfrac{1}{2}[U^T(\xi,\eta,\zeta) + U^T(\xi,-\eta,\zeta)]\ ;$$

$$V_S^T(\xi,\eta,\zeta) = \tfrac{1}{2}[V^T(\xi,\eta,\zeta) - V^T(\xi,-\eta,\zeta)]\ ;$$

$$W_S^T(\xi,\eta,\zeta) = \tfrac{1}{2}[W^T(\xi,\eta,\zeta) + W^T(\xi,-\eta,\zeta)]\ .$$

Bearing in mind the Prandtl-Glauert transformation, and taking (4) into account, we obtain a correction (for the symmetrical part) to the pressure on the surface of the body

$$c_{p_s}^i(X,0,Z) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-L}^{L}\left\{\frac{\beta^2 H U_s^T(\xi,H,\zeta) - V_s^T(\xi,H,\zeta)(X-\xi)}{[(X-\xi)^2 + \beta^2 H^2 + \beta^2(Z-\zeta)^2]^{3/2}}\right\} d\xi d\zeta -$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{H}\left\{\frac{-\beta^2 U_s^T(\xi,\eta,L)(Z-L) - W_s^T(\xi,\eta,L)(X-\xi)}{[(X-\xi)^2 + \beta^2\eta^2 + \beta^2(Z-L)^2]^{3/2}}\right\} d\xi d\eta -$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{H}\left\{\frac{\beta^2 U_s^T(\xi,\eta,L)(Z+L) - W_s^T(\xi,\eta,L)(X-\xi)}{[(X-\xi)^2 + \beta^2\eta^2 + \beta^2(Z-L)^2]^{3/2}}\right\} d\xi d\eta\ . \qquad (5)$$

At transition to cylindrical coordinates for axisymmetrical flow from (5) we obtain an expression for the correction to the coefficient of pressure, as was shown in paper (5)

$$c_p^i(X,0) = -\beta^2 R^2\int_{-\infty}^{\infty}\frac{U^T(\xi,R)d\xi}{[(X-\xi)^2 + \beta^2 R^2]^{3/2}} + R\int_{-\infty}^{\infty}\frac{V^T(\xi,R)(X-\xi)d\xi}{[(X-\xi)^2 + \beta^2 R^2]^{3/2}}\ ,$$

where R is the radius of the cylindrical control surface.

Similarly, in the limit as $L \to \infty$ for the integration with respect to ζ and in the case of components of velocity independent of this variable, we obtain from (5)

$$c_p^i(x,0) = -\frac{\beta H}{\pi}\int_{-\infty}^{\infty}\frac{U_s^T(\xi,H)d\xi}{(X-\xi)^2 + \beta^2 H^2} + \frac{1}{\pi\beta}\int_{-\infty}^{\infty}\frac{V_s^T(\xi,H)(X-\xi)d\xi}{(X-\xi)^2 + \beta^2 H^2}\ ,$$

which corresponds to the result of papers (4,6).

3 The anti-symmetrical potential $\Phi_A(x,y,z)$ satisfies the following boundary conditions:

(a) on the surface of the body the condition of no-flow, referred to the plane xz, $y = 0$

$$\left.\frac{\partial\Phi_A(x,\pm 0,z)}{\partial y}\right|_{(x,z)\in S_{wing})} = \frac{f'_x(x,z)}{\beta} ;$$

(b) (the condition is also referred to the plane xz, $y = 0$) – continuity of pressure across on the trailing-vortex sheet leads to

$$\frac{\partial\Phi_A(x,+0,z)}{\partial x} = \frac{\partial\Phi_A(x,-0,z)}{\partial x} ,$$

and consequently

$$\frac{\partial\Phi_A(x,\pm 0,z)}{\partial x} = 0 ;$$

the continuity of the normal component of velocity on the trailing-vortex sheet implies

$$\frac{\partial\Phi_A(x,+0,z)}{\partial y} = \frac{\partial\Phi_A(x,-0,z)}{\partial y} ;$$

(c) on the surface S $\frac{\partial\Phi_A}{\partial x}$ and $\frac{\partial\Phi_A}{\partial n}$ are known. Similarly for the symmetrical potential

$$\Phi_A^T(x,y,z) = \frac{1}{4\pi}\int\limits_{S_{wing}}\int\limits_{+S_n}\left\{\Phi_A(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_A(\xi,\eta,\zeta)}{\partial n}\frac{1}{r}\right\} d\xi d\zeta +$$

$$+\frac{1}{4\pi}\iint\limits_S\left\{\Phi_A^T(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_A^T(\xi,\eta,\zeta)}{\partial n}\cdot\frac{1}{r}\right\} dS . \quad (6)$$

The integral over the surface of the wing and the vortex sheet is transformed in accordance with anti-symmetry to the form

$$\frac{1}{4\pi}\int\limits_{S_{wing}}\int\limits_{+S_n}\left\{\Phi_A(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial\Phi_A(\xi,\eta,\zeta)}{\partial n}\cdot\frac{1}{r}\right\} d\xi d\zeta =$$

$$= \frac{1}{2\pi}\iint\limits_{S_{wing}}\frac{\partial\Phi_A(\xi,0,\zeta)}{\partial\xi}\cdot\frac{y}{y^2 + (z-\xi)^2}\left(1 + \frac{x-\xi}{r}\right) d\xi d\zeta . \quad (7)$$

In the case of unlimited flow, the integrals for the bounding surface vanish leaving for the potential the following expression

$$\Phi_A^F(x,y,z) = \frac{1}{2\pi} \iint\limits_{S_{wing}} \frac{\partial \Phi_A^F(\xi,0,\zeta)}{\partial \xi} \, \frac{y}{y^2 + (z-\xi)^2} \left(1 + \frac{x-\xi}{r}\right) d\xi d\zeta \, . \qquad (8)$$

Taking into account (6) to (8) we have for the wall-induced potential .

$$\Phi_A^i(x,y,z) = \Phi_A^T(x,y,z) - \Phi_A^F(x,y,z) =$$

$$= \frac{1}{2\pi} \iint\limits_{S_{wing}} \frac{\partial \Phi_A^i(\xi,0,\zeta)}{\partial \xi} \, \frac{y}{y^2 + (z-\xi)^2} \left(1 + \frac{x-\xi}{r}\right) d\xi d\zeta \; +$$

$$= \frac{1}{4\pi} \iint\limits_{S} \left\{ \Phi_A^T(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial \Phi_A^T(\xi,\eta,\zeta)}{\partial n} \cdot \frac{1}{r} \right\} dS \, . \qquad (9)$$

From the condition of no-flow

$$\left. \frac{\partial \Phi_A^i(x,\pm 0,z)}{\partial y} \right|_{(x,z)(\in S_{wing})} = \frac{\partial}{\partial y}\left[\Phi_A^T(x,\pm 0,z) - \Phi_A^F(x,\pm 0,z)\right] \Bigg|_{(x,z)\in S} = 0$$

and taking (8) and (9) into account we obtain

$$\frac{\text{p.v.}}{2\pi} \iint\limits_{S_{wing}} \frac{\partial \Phi_A^i(\xi,0,\zeta)_2}{\partial \xi} \cdot \frac{1}{(z-\zeta)^2} \left(1 + \frac{x-\xi}{\sqrt{(x-\xi)^2 + (z-\zeta)^2}}\right) d\xi d\zeta \; -$$

$$- \frac{1}{4\pi}\frac{\partial}{\partial y} \left\{ \iint\limits_{S} \left[\Phi_A^T(\xi,\eta,\zeta)\left(\frac{\partial}{\partial n}\frac{1}{r}\right) - \frac{\partial \Phi_A^T(\xi,\eta,\zeta)}{\partial n} \cdot \frac{1}{r}\right] dS \right\} \Bigg|_{y=0} = 0 \, .$$

(p.v. appears to represent 'principal value' but how it is defined is not clear. Editor)

Performing the integration by parts in the second term of the equation obtained and representing the integral for the surface S as for a working section of rectangular section (see Fig 2), taking anti-symmetry into account, we arrive at an equation which determines the anti-symmetrical part of the wall-induced potential.

$$\frac{\underline{\text{p.v.}}}{2\pi} \iint\limits_{S_{wing}} \frac{\partial \Phi_A^i(\xi,+0,\zeta)}{\partial \xi} \cdot \frac{1}{(z-\zeta)^2} \left(1 + \frac{x-\xi}{\sqrt{(x \quad \xi)^2 + (z-\zeta)^2}} \right) d\xi d\zeta =$$

$$= \frac{1}{2\pi} \iint\limits_{S_1} \left\{ U_A^I(\xi,H,\zeta) \left[\frac{H^2 - (z-\zeta)^2}{[H^2 + (z-\zeta)^2]^2} \left(1 + \frac{x-\xi}{\sqrt{(x-\xi)^2 + H^2 + (z-\zeta)^2}} \right) + \right. \right.$$

$$\left. + \frac{H^2(x-\xi)}{[H^2 + (z-\xi)^2]\,[(x-\xi)^2 + H^2 + (z-\zeta)^2]^{3/2}} \right] +$$

$$\left. + \frac{V_A^T(\xi,H,\zeta)H}{[(x-\xi)^2 + H^2 + (z-\zeta)^2]^{3/2}} \right\} d\xi d\zeta +$$

$$+ \frac{1}{2\pi} \iint\limits_{\frac{1}{2}S_3} \left\{ U_A^T(\xi,\eta,L) \left[\frac{-2\eta(z-L)}{[\eta^2 + (z-L)^2]^2} \times \left(1 + \frac{x-\xi}{\sqrt{(x-\xi)^2 + \eta^2 + (z-L)^2}} \right) - \right. \right.$$

$$\left. - \frac{(z-L)(x-\xi)\,.\,\eta}{[\eta^2 + (z-L)^2]\,[x-\xi^2 + \eta^2 + (z-L)^2]^{3/2}} \right] +$$

$$\left. + \frac{W_A^T(\xi,\eta,L)\,.\,\eta}{[(x-\xi)^2 + \eta^2 + (z-L)^2]^{3/2}} \right\} d\xi d\eta +$$

$$+ \frac{1}{2\pi} \iint\limits_{\frac{1}{2}S_4} \left\{ U_A^T(\xi,\eta,L) \left[\frac{2\eta(z+L)}{[\eta^2 + (z+L)^2]^2} \times \left(1 + \frac{x-\xi}{\sqrt{(x-\xi)^2 + \eta^2 + (z+L)^2}} \right) + \right. \right.$$

$$\left. + \frac{(z+L)(x-\xi)\,.\,\eta}{[\eta^2 + (z+L)^2]\,[x-\xi^2 + \eta^2 - (z-L)^2]^{3/2}} \right] +$$

$$\left. + \frac{W_A^T(\xi,\eta,L)\,.\,\eta}{[(x-\xi)^2 + \eta^2 + (z+L)^2]^{3/2}} \right\} d\xi d\eta\,.$$

The anti-symmetrical components of velocity on the surface S are determined in the following manner:

$$U_A^T(\xi,\eta,\zeta) = \tfrac{1}{2}[U^T(\xi,\eta,\zeta) - U^T(\xi,-\eta,\zeta)]\,;$$

$$V_A^T(\xi,\eta,\zeta) = \tfrac{1}{2}[V^T(\xi,\eta,\zeta) + V^T(\xi,-\eta,\zeta)]\,;$$

$$W_A^T(\xi,\eta,\zeta) = \tfrac{1}{2}[W^T(\xi,\eta,\zeta) - W^T(\xi,-\eta,\zeta)]\,.$$

The right-hand part (we denote it $F(x,z)$) of the equation obtained is the downwash of the flow induced by the walls of the tunnel in the region of location of the model. The magnitude $\dfrac{\partial \Phi_A^i}{\partial \xi}$ represents the intensity of the vortex doublets, which satisfy the condition of no-flow on the wing in the presence of walls.

The equation obtained may be interpreted as an equation of the potential of flow near the wing.

For a compressible gas, after the Prandtl-Glauert transformation in equation (10) and taking into account that $c_p^i = 2\dfrac{\partial \Phi_A^i}{\partial \xi}$, we obtain

$$-\frac{\text{p.v.}}{4\pi}\iint\limits_{S_{wing}} c^i_{p_A}(\xi,+0,\zeta)\,\frac{1}{(Z-\zeta)^2}\left[\frac{X-\xi}{\sqrt{(X-\xi)^2+\beta^2(Z-\zeta)^2}}\right]d\xi d\zeta = F_{comp}(X,Z)$$

$$F_{comp}(X,Z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-L}^{L}\left\{U_A^T\left[\frac{H^2-(Z-\zeta)^2}{\beta[H^2+(Z-\zeta)^2]^2}\left(1+\frac{X-\xi}{\sqrt{(X-\xi)^2+\beta^2H^2+\beta^2(Z-\zeta)^2}}\right)+\right.\right.$$

$$\left.+\frac{\beta H^2(X-\xi)}{[H^2+(Z-\zeta)^2]\,[(X-\xi)^2+\beta^2H^2+\beta^2(Z-\zeta)^2]^{3/2}}\right]+$$

$$\left.+\frac{V_A^T(\xi,\eta,\zeta)\beta H}{[(X-\zeta)^2+\beta^2H^2+\beta^2(Z-\zeta)^2]^{3/2}}\right\}d\xi d\zeta +$$

$$+\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{0}^{H}\left\{U_A^T(\xi,\eta,L)\left[\frac{-2\eta(Z-L)}{\beta[\eta^2+(Z-L)^2]^2}\left(1+\frac{(X-\xi)}{\sqrt{(X-\xi)^2+\beta^2\eta^2+\beta(Z-L)^2}}\right)-\right.\right.$$

$$-\frac{(Z-L)(X-\xi)\beta\eta}{[\eta^2+(Z-L)^2]\;[(X-\xi)^2+\beta^2\eta^2+\beta^2(Z-L)^2]^{3/2}}+$$

$$+\frac{2\eta(Z+L)}{\beta[\eta^2+(Z+L)^2]^2}\left(1+\frac{(X-\xi)}{\sqrt{(X-\xi)^2+\beta^2\eta^2+\beta^2(Z+L)^2}}\right)+$$

$$\left.-\frac{(Z+L)(X+\xi)\beta\eta}{[\eta^2(Z+L)^2]\;[(X-\xi)^2+\beta^2\eta^2+\beta^2(Z+L)]^{3/2}}\right]+$$

$$+W_A^T(\xi,\eta,L)\left[\frac{\beta\eta}{[(X-\xi)^2+\beta^2\eta^2+\beta^2(Z-L)^2]^{3/2}}\right]+$$

$$\left.+\left[\frac{\beta\eta}{[(X-\xi)^2+\beta^2\eta^2+\beta^2(Z+L)^2]^{3/2}}\right]\right\}d\xi d\eta\ , \qquad \text{..... (11)}$$

where $F_{comp}(X,Z)$ is the downwash in the flow of a compressible gas.

As a whole, the correction to the pressure coefficient on the surface of the model investigated in the wind-tunnel, assumes the form

$$c_p^i(X,Z)\Big|_{Y=\pm 0} = c_{PS}^i = c_{PS}^j(X,Z) \pm c_{P_A}^i(X,Z).$$

4 Formula (5) which defines the symmetrical part of the correction to the pressure coefficient, and formula (11) for downwash of the flow in the numerical calculation are represented in approximate form

$$F_{comp}(X,Z) = \frac{1}{2\pi}\sum_{i=1}^{N_x}\sum_{j=1}^{N_z}\left(U_{A_{i,j}}^T\cdot A_{i,j}+V_{A_{i,j}}^T\cdot B_{i,j}\right)+$$

$$+\frac{1}{2\pi}\sum_{k=1}^{L_x}\sum_{l=1}^{L_Y}\left(U_{A_{k,l}}^T\cdot D_{k,l}+W_{A_{k,l}}^T\cdot C_{k,l}\right)\ ; \qquad (12)$$

$$c^{i}_{p_s}(X,Z) = \frac{1}{\pi}\sum_{i=1}^{N_x}\sum_{j=1}^{N_z}\left(U^T_{s_{i,j}} \cdot A'_{i,j} + V^T_{s_{i,j}} \cdot B'_{i,j}\right) +$$

$$+\frac{1}{\pi}\sum_{k=1}^{L_x}\sum_{l=1}^{L_Y}\left(U^T_{s_{k,l}} \cdot D'_{k,l} + W^T_{s_{k,l}} \cdot C'_{k,l}\right) , \qquad (13)$$

where

$$A_{i,j} = \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{n+m}\left[-\xi_{i+m}(Z-\zeta_{j-n})/G_{j-n}\right] +$$

$$+ (Z-\zeta_{j+n})R_{i-m,j+n}/G_{j+n} - \frac{\beta}{2}\ln\left|\frac{R_{i+m,j+n}-(Z-\zeta_{j+n})}{R_{i-m,j+n}-(Z-\zeta_{j+n})}\right| ;$$

$$B_{i,j} = \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{n+m}\operatorname{arctg}\left[\frac{(Z-\zeta_{j+n})(X-\xi_{j+m})}{H \cdot R_{i+m,j+n}}\right] ;$$

$$D_{k,l} = \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{n+m}\left\{\xi_{k+m}\left[(Z-L)/P^1_l - (Z+L)\,P^2_l\right] -\right.$$

$$\left. -(Z-L)Q^1_{k+m,l+m}/P^1_l + (Z+L)Q^2_{k+m,l+n}\,P^2_l\right\} ;$$

$$C_{k,l} = \sum_{m=0}^{1}\sum_{n=0}^{1}\frac{(-1)^{n+m}}{2}\left[\ln\left|\frac{Q^1_{k+m,l+m}-(X-\xi_{k+m})}{Q^1_{k+m,l+m}-(X-\xi_{k+m})}\right| +\right.$$

$$\left. + \ln\left|\frac{Q^2_{k+m,l+m}+(X-\xi_{k-m})}{Q^2_{k+m,l+m}-(X-\xi_{k-m})}\right|\right] .$$

$$A'_{i,j} = \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{m+n}\operatorname{arctg}\left[\frac{(Z-\zeta_{j+n})(X-\xi_{i+m})}{HR_{i-m,j-n}}\right] ;$$

$$B'_{i,j} = \sum_{m=0}^{1}\sum_{n=0}^{1}(-1)^{m+n}\ln\left|(Z-\zeta_{j-n})\beta - R_{i-m,j-n}\right| ;$$

$$D'_{k,l} = \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{m+n} \left\{ \operatorname{arctg} \left[\frac{\eta_{l-n}(X - \xi_{k-m})}{(Z - L)Q^1_{k+m,l-n}} \right] - \right.$$

$$\left. - \operatorname{arctg} \left[\frac{\eta_{l+n}(X - \xi_{k+m})}{(Z + L)Q^2_{k+m,l+n}} \right] \right\} ;$$

$$C'_{k,l} = \sum_{m=0}^{1} \sum_{n=0}^{1} (-1)^{m+n} \left\{ \ln \left| \beta\eta_{l+n} + Q^1_{k+m,l+n} \right| + \ln \left| \beta\eta_{l+n} + Q^2_{k+m,l+n} \right| \right\} ;$$

$$G_j = H^2 + (Z - \zeta_j)^2 ;$$

$$R_{ij} = \sqrt{(X - \xi_i)^2 + \beta^2 H^2 + \beta^2 (Z - \xi_j)^2} ;$$

$$P^1_l = \eta_l^2 + (Z - L)^2 ;$$

$$P^2_l = \eta_l^2 + (Z + L)^2 ;$$

$$Q^1_{k,l} = \sqrt{(X - \xi_k)^2 + \beta^2 \eta_l^2 + \beta^2 (Z - L)^2} ;$$

$$Q^2_{k,l} = \sqrt{(X - \xi_k)^2 + \beta^2 \eta_l^2 + \beta^2 (Z + L)^2} .$$

N_x, N_z and L_x, L_y are the number of sub-divisions on the horizontal and lateral wall respectively (see Fig 2), the values of the components of velocity are taken in the centres of the rectangular sections. In the analysis of expressions (12) and (13) the question arises of the quantity of sub-divisions and the dimensions of the region on which the sums are calculated.

It is seen directly from equations (5) and (11) that the region of integration around the points in question may be truncated, since the sub-integral expressions tend to zero as $\frac{1}{r}$ at $r \to \infty$.

It was shown in paper (7) that for linear boundary conditions (Darcy type) perturbations from the walls may be represented by discrete features. The flow field induced by the walls, was modelled by a combination of horseshoe vortices and sources, without going into details of what porosity the walls possess. By performing an investigation of the accuracy of the integration and by virtue of the linearity of the flow, it is

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possible to estimate the necessary quantity of sub-divisions of the surface of the tunnel so as to obtain satisfactory results both in the case of arbitrary models and boundary conditions.

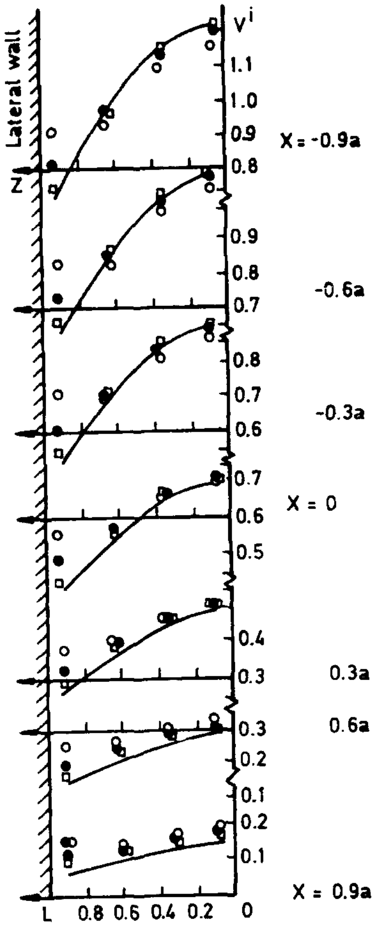
Let us examine a working section, having a rectangular section of half-width $L = 1.5$ and a semi-height $H = 0.86L$. The central horseshoe vortex of semi-span $a = \frac{2}{3}L$ is located such that the vortex filaments extend to infinity parallel to the axis X from points having the coordinates (0,0,-a) and (0,0,a).

The disposition of the four horseshoe vortices, which represent the perturbation from the walls is determined by mirror imaging of the central vortex from the lateral walls.

In Fig 3 for the perturbation modelled by four horseshoe vortices in sections along the X-axis there are shown for comparison both the calculated and 'exact' values of the downwash of the flow with varying sub-divisions in the process of integration. A satisfactory agreement in the region of the location of the model is already attained for $X1 = -2a$ and $X2 = 2a$ (coordinates of the beginning and end of the integration) and $N_x = L_x = 10$, $N_y = 2$, $N_z = 3$. Fig 4 illustrates the calculation of the symmetrical correction from a perturbation of a source-sink type. In the case of direct measurement of the component of velocity on the walls, it is necessary to allow for the accuracy of the fixing instrument. An approximate model of the measurement instrument was selected for estimating the accuracy.

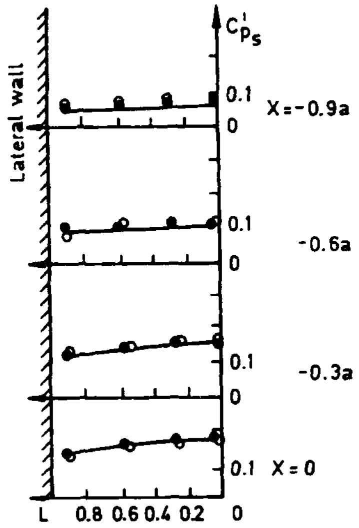
The magnitude of the measured parameter $U = U_0 + bS'$, where U_0 is the precise value of the magnitude, b is a constant, S' is a random magnitude generated by the operator of the random numbers $0 < S' < 1$. If it is assumed that the parameter of porosity is measured by the same instrument, then for the magnitude $V = kU$ we have $V = V_0 + 2bS'$. For the determinations of b on Fig 5 there are shown the distributions of the components measured by the proposed instrument of the axial velocity on the upper wall.

The mean-quadratic deviation amounted to $\sigma = 6.9\%$ and $\sigma = 3.4\%$. On Fig 6 for sub-division of the walls into sections, as demonstrated on Fig 2 there are given the results which illustrate the influence of the mean-quadratic error of measurement of the parameters of flow on the mean-quadratic error in calculation of the downwash of the flow and of the axial component of velocity in various sections along Z . In the case of error of measurement of the parameters on the control surface of $\sigma_v \leq 5\%$, the error for the calculated magnitudes of wall-induced velocity amounts to $\sigma \lesssim 10\%$.



□ $X1 = -3a$; $X2 = 3a$, $L_x = N_x = 20$; $L_y = 5$; $N_z = 8$
● $-3a$; $3a$; 10; 3; 5
○ $2a$; $2a$; 5; 2; 5

Fig 3



○ $X1 = -2a$; $X2 = 2a$; $N_x = 5$; $L_y = 2$; $N_z = 5$

● $X1 = -3a$; $X2 = 3a$; $N_x = 10$; $L_y = 3$; $N_z = 5$

Fig 4

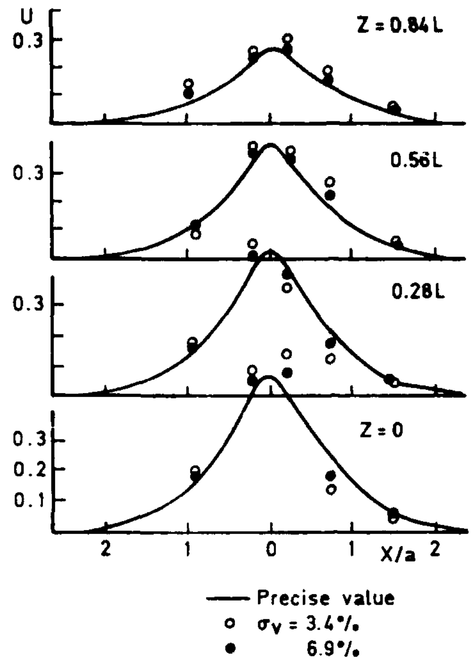

U
0.3
Z = 0.84 L
0.3
0.56L
0.3
0.28L
0.3
0.2
0.1
Z = 0
2
1
0
1
X/a
2
Precise value
σv = 3.4%
6.9%

Fig 5

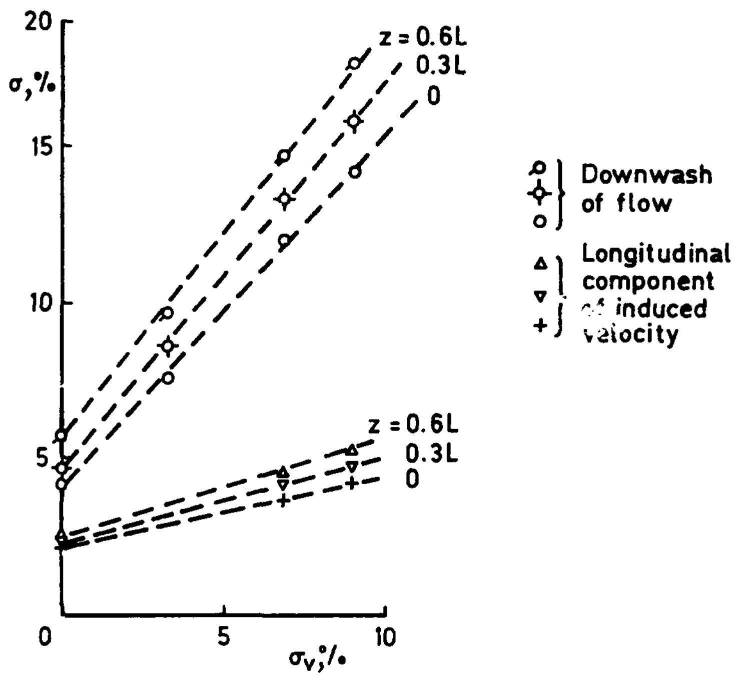

20
σ,%
15
10
5
0
5
10
σv,%
z = 0.6L
0.3L
0
z = 0.6L
0.3L
0
Downwash
of flow
Longitudinal
component
of induced
velocity

Fig 6

REFERENCES

No.	Author	Title, etc
1	–	The effect of walls in transonic tunnels and ways of diminishing it. *Trans. TsAGI*, No.2028 (1980)
2	A.P. Byrkin I.I. Mezhirov	The numerical investigation of the induction of porous walls of the working part of a low-velocity wind-tunnel. *Uch. Zap. TsAGI*, No.6, **8** (1977)
3	J. Vayssaire	Survey of methods for correcting wall constraints in transonic wind tunnels. *NN (AGARD R-601)* (1973)
4	C.F. Lo	Tunnel interference assessment by boundary measurements. *AIAA Journal*, 4 (1972)
5	S.A. Glaskov V.M. Ivanova	Investigation of the induction of porous walls of a wind-tunnel from known parameters of flow near them. *Uch. Zap. TsAGI*, No.4, **13** (1982)
6	E.M. Kraft J.A. Dahm	Direct assessment of wall interference in a two-dimensional subsonic wind-tunnel. *AIAA Journal*, 82-0187, 11-14 January 1982
7	A.V. Semenov O.K. Chikina	*The effect of porous walls on a horseshoe vortex subjected to incompressible flow.* TsAGI (unpublished)
8	A.I. Ivanov	On the calculation of the boundary condition on the porous walls of wind-tunnels. *Uch. Zap. TsAGI*, No.1, **16** (1985)
9	A.I. Ivanov V.M. Neiland A.V. Semenov O.K. Semenova	*Calculated and experimental investigation of the interference of models with porous walls and a tail support during tests in a sub-sonic flow.* Report to the Soviet-French Sub-Group on Aerodynamics, Aviation Acoustics and Strength, M. (1984)